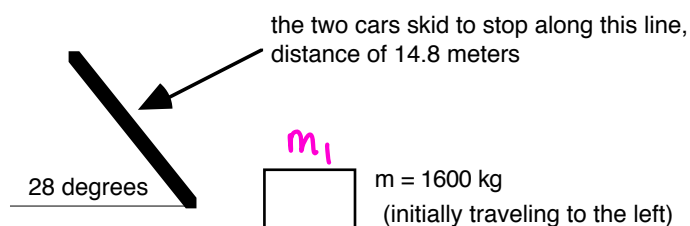
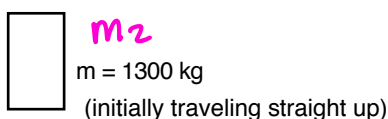
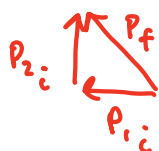


## 2D Momentum Problems

1. Two cars approach a 90° intersection. Neither driver is paying attention to what they are doing, and they collide. After the collision, the cars stick together, and skid to a stop in 14.8 meters at an angle as shown. The two cars have masses of 1300 kg and 1600 kg, as shown. You happen to know that the coefficient of friction between the tires and the road was  $\mu = 0.3$ . How fast were the drivers going just prior to the collision?



$$\Sigma \vec{p}_i = \Sigma \vec{p}_f$$



The final speed after the collision is the initial speed of the skid;

Find  $v_i$  skid

$$v_f = 0 \quad d = 14.8 \text{ m} \quad \mu = 0.3$$

$$\frac{1}{2} m v_i^2 - \mu m g d = 0$$

$$v_i^2 = 2 \mu g d = 2(0.3)(10)(14.8)$$

$$v_i = 9.42 \text{ m/s}$$

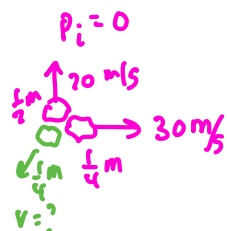
$$27,318 \cos 25 = 1600 v_1$$

$$v_1 = 15.5 \text{ m/s}$$

$$27,318 \sin 25 = 1300 v_2$$

$$v_2 = 8.9 \text{ m/s}$$

2. A 5 kg object initially at rest explodes into three (unequal) pieces:  $\frac{1}{4}$  of the mass has a velocity of  $30\hat{i}$  m/s after the explosion while  $\frac{1}{2}$  of the mass has a velocity of  $20\hat{j}$  m/s. What is the speed of the third mass after the explosion?



$$\Sigma \vec{p}_i = \Sigma \vec{p}_f$$

$$0 = (\frac{1}{4}m)30\hat{i} + (\frac{1}{2}m)20\hat{j} + (\frac{1}{4}m)\vec{v}$$

$$0 = 30\hat{i} + 40\hat{j} + \vec{v}$$

$$\vec{v} = -30\hat{i} - 40\hat{j} \text{ m/s}$$

$$So \quad v = \sqrt{(-30)^2 + (-40)^2}$$

$$v = 50 \text{ m/s}$$

3. An object has some initial velocity when it explodes into two pieces.  $\frac{1}{3}$  of the mass has a velocity of 5 m/s at  $25^\circ$  after the explosion. The other  $\frac{2}{3}$  of the mass has a velocity of 4 m/s at  $-50^\circ$ . ( $0^\circ$  would be in the  $+\hat{i}$  direction.) What was the initial velocity of the whole mass?

$$\Sigma \vec{p}_i = \Sigma \vec{p}_f$$

$$m\vec{v}_i = (\frac{1}{3}m)5(\cos 25\hat{i} + \sin 25\hat{j}) + (\frac{2}{3}m)(4)(\cos 50\hat{i} - \sin 50\hat{j})$$

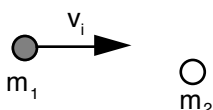
$$\vec{v}_i = 1.51\hat{i} + 0.704\hat{j} + 1.71\hat{i} - 2.04\hat{j}$$

$$\vec{v}_i = 3.2\hat{i} - 1.3\hat{j} \text{ m/s}$$

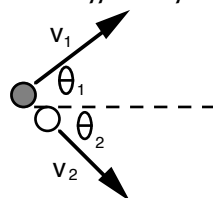
## 2D Momentum Problems

The 4 to 6 all use the same diagram (but the numbers are different for each problem.)

pre-collision



post-collision



4. A ball of mass 0.2 kg is traveling horizontally with a speed of 3.5 m/s. It collides with a ball of 0.4 kg, initially at rest. After the collision, the first ball is traveling with a velocity of 3 m/s at an angle of  $30^\circ$  above the original direction of motion. What is the velocity of the other ball?

$$m_1 = 0.2 \text{ kg}$$

$$v_i = 3.5 \text{ m/s}$$

$$m_2 = 0.4 \text{ kg}$$

$$v_1 = 3 \text{ m/s}$$

$$\theta_1 = 30^\circ$$

$$\hat{i}) m_1 v_i = m_1 v_1 \cos \theta_1 + m_2 v_2 \cos \theta_2$$

$$(0.2)(3.5) = (0.2)(3) \cos 30 + (0.4) v_2 \cos \theta_2$$

$$0.7 = 0.52 + 0.4 v_{2x}$$

$$0.4 v_{2x} = 0.18$$

$$\rightarrow v_{2x} = 0.45 \text{ m/s}$$

$$\hat{j}) m_1 v_i \sin \theta_1 = m_2 v_2 \sin \theta_2$$

$$(0.2)(3) \sin 30 = (0.4) v_2 \sin \theta_2$$

$$0.3 = 0.4 v_{2y}$$

$$v_{2y} = 0.75 \text{ m/s (but } \downarrow, \text{ so } -)$$

$$\text{So } \vec{v}_2 = 0.45 \hat{i} - 0.75 \hat{j} \text{ m/s}$$

5.  $m_1 = 0.25 \text{ kg}$  and  $m_2 = 0.15 \text{ kg}$ . The speeds after the collision are  $v_1 = 4 \text{ m/s}$  and  $v_2 = 6 \text{ m/s}$ . If  $\theta_1 = 30^\circ$ .

a. Was the collision elastic? (Hint: Find the initial speed first.)

$$\hat{i}) (0.25) v_i = (0.25)(4) \cos 30 + (0.15)(6) \cos \theta_2$$

$$v_i = 3.46 + 3.6 \cos \theta_2$$

$$\hat{j}) (0.25)(4) \sin 30 = (0.15)(6) \sin \theta_2$$

$$1.5 = 0.9 \sin \theta_2$$

$$\sin \theta_2 = 0.556$$

$$\theta_2 = 33.75^\circ$$

$$\text{So } v_i = 3.46 + 3.6 \cos(33.75) \quad v_i = 6.45 \text{ m/s}$$

$$\text{So } K_i = \frac{1}{2} (0.25) (6.45)^2$$

$$K_i = 5.2 \text{ J}$$

$$K_f = \frac{1}{2} (0.25) (4)^2 + \frac{1}{2} (0.15) (6)^2$$

$$K_f = 2 + 2.7$$

$$K_f = 4.7 \text{ J}$$

Not elastic; lost 0.5 J

b. What was the impulse on  $m_1$ ?

Because the impulses are  $\pm$  opposite, let's find the impulse on  $m_2$  first!

$$\text{So } J_2 = \Delta p_2 = (0.15)(6 \cos 33.75 \hat{i} - 6 \sin 33.75 \hat{j})$$

$$= 0.75 \hat{i} - 0.5 \hat{j} \text{ Ns}$$

$$\therefore J_1 = -0.75 \hat{i} + 0.5 \hat{j} \text{ Ns}$$

## 2D Momentum Problems

6. Both masses are 0.25 kg. The initial speed of  $m_1$  was 5 m/s and the collision was elastic. If  $\theta_2 = 30^\circ$ , how fast were the masses going after the collision?

Since  $m_1 = m_2$  & elastic,  $\theta_1 + \theta_2 = 90^\circ \rightarrow$  So  $\theta_1 = 60^\circ$

$$\hat{i}) \quad m_1 v_i = m_1 v_1 \cos \theta_1 + m_1 v_2 \cos \theta_2$$

$$5 = v_1 \cos 60 + v_2 \cos 30$$

$$\hat{j}) \quad m_1 v_i \sin \theta_1 = m_1 v_2 \sin \theta_2$$

$$v_1 \sin 60 = v_2 \sin 30 \rightarrow \underline{v_1 = (.577) v_2}$$

$$\text{So } 5 = (.577) v_2 \cos 60 + v_2 \cos 30$$

$$5 = .289 v_2 + .866 v_2 = 1.16 v_2$$

$$\boxed{v_2 = 4.33 \text{ m/s}}$$

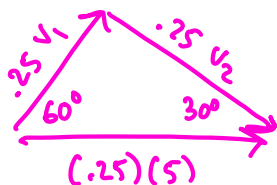
$$\text{So } v_1 = (.577)(4.33)$$

$$\boxed{v_1 = 2.50 \text{ m/s}}$$

Answers:

- 1) 1600 kg: 15 m/s; 1300 kg: 9.9 m/s    2) 50 m/s    3)  $3.23i - 1.34j$  m/s  
 4)  $0.45i - 0.75j$  m/s    5. a) 6.5 m/s; not elastic    b)  $-0.76i + 0.5j$  Ns  
 6)  $v_1 = 2.5$  m/s;  $v_2 = 4.3$  m/s

Also



$$v_1 = 5 \sin 30 = 2.5 \text{ m/s}$$

$$v_2 = 5 \sin 60 = 4.33 \text{ m/s}$$

$$\left( \begin{array}{l} \text{or } v_1 = 5 \cos 60 \\ \text{ \& } v_2 = 5 \cos 30 \end{array} \right)$$